Shahid Chamran University of Ahvaz

Journal of Advanced Mathematical Modeling
ISSN (print): 2251-8088, ISSN (on-line): 2645-6141
Vol. 13, No. 5 (2024), pp. 1-22.
DOI: 10.22055/JAMM.2023.42206.2108

MINIMIZATION OF HIV INFECTION AMONG NIGERIAN WOMEN THROUGH THE USE OF MICROBICIDES: AN INSIGHT FROM MATHEMATICAL MODELING

ABAYOMI A. AYOADE*, SAMUEL T. AKINYEMI AND T. OYEDEPO

Communicated by: A. R. Fakharzadeh

Abstract. A recent report indicated that one hundred and thirty thousand Nigerians were lately infected with HIV with the majority of infections resulting from unguarded vaginal sex. About two-thirds of new HIV infections in adults exist in women. Male circumcision and male condoms limit the danger of HIV infection, but the adoption of these procedures is beyond the control of Nigerian women due to gender inequality and gender-based violence against women in the country. In an attempt to provide a mathematical framework to examine a potential female-controlled strategy of HIV acquisition in Nigeria, a mathematical model is modified and analyzed for the transmission of HIV by incorporating pre-exposure prophylaxis (PrEP) in the form of a microbicide. The solutions of the model are proved to be positive. The critical points of the model and the epidemic threshold known as the reproduction number are also derived. The restricted case of total compliance to the use of the microbicide is analyzed by proving the global stability of the disease-free equilibrium (DFE) of the model. The general case which permits individuals to default the microbicide is also investigated by proving the global stability of the endemic equilibrium (EE) of the model. Numerical simulation is carried out to verify the analytical results and the results of the simulation show that strict compliance and consistent use of the microbicide may tend HIV acquisition among Nigerian women to zero.

MSC(2020): 92B05, 92D30, 34D20.

Keywords: HIV, Model, Microbicide, Reproduction number, Simulation.

Received: 29 October 2022, Accepted: 18 July 2023.

*Corresponding author.

1. Introduction

For nearly four decades, HIV/AIDS, with about forty million infections and seven hundred and seventy thousand deaths globally as of 2018, has been a major source of threat to human existence [35]. Despite significant success in HIV treatments, eradication of HIV/AIDS remained elusive, and approximately two million people were newly infected across the globe in 2018 [28]. Therefore, the prevention of new infections becomes an enormous task. About two-thirds of the individuals living with HIV in sub-Saharan Africa are women. The figure is attributed to gender inequality, under-age marriage, forced marriage, rape and other sexual violence against women in the region [29]. Nigeria accounts for the second-highest HIV epidemic worldwide with around 3.1 million of her total population living with the virus in 2017 and 130 000 new HIV infections in 2018 [22, 30]. The current estimates indicate that there are over 1.9 million individuals infected with HIV in Nigeria and the present prevalence of the disease among adults (15-49 years) is 1.4% [31]. Many individuals infected with HIV in Nigeria are unaware of their HIV status. For example, the Punch reported that 10 000 unidentified individuals were living with HIV in the Imo state of Nigeria [26]. Besides, the country has not been able to meet up with the suggested figure of HIV testing and counseling sites [11]. United Nations Program on HIV/AIDS (UNAIDS) reported that about two-thirds of new HIV infections in Central and West Africa emanated from Nigeria [32]. The country, together with Uganda and South Africa, is responsible for about half of every new HIV infection in sub-Saharan Africa yearly despite the achievement of a five percent decrease in new HIV infections between 2010 and 2017 [33].

Unhealthy heterosexual behavior accounts for about 80% of all new HIV infections in Nigeria, with the larger part of the remaining 20% occurring in major particular populations of individuals such as sex workers. Worldwide, female prostitutes are more than 13.5 times as likely to be infected with HIV as women in the general human population [18]. Six states make up 41% of individuals living with HIV in Nigeria, including Benue, Kaduna, Lagos, Akwa-Ibom, Kano, and Oyo [22]. While HIV prevalence among adults (15-49 years) is higher in some zones in Nigeria like the South-South zone with 3.1%, the North-Central zone with 2.0% and the South-East zone with 1.9%, it is lower in other zones like the North-East zone, the South-West zone and the North-West zone with 1.1%, 1.1% and 0.6% respectively [30]. The incidence of the disease is higher in rural areas

(4%) than in urban areas (3%) [22]. The AIDS-related illnesses mortality fell from 150 000 in 2017 to 53 000 in 2019 [31]. However, poor access to antiretroviral therapy (ART) remain a major problem for individuals living with HIV, implying that there exists many AIDS-related deaths that are unaccounted for in Nigeria [7].

Many Nigerian women, because of narrow economic choices and gender inequality, cannot negotiate sexual encounters, leaving them at risk of unplanned pregnancy and sexually transmitted infections (STIs). There is an enormous difference between the figures of men and women who are infected with HIV in Nigeria: of the 1.8 million people living with HIV, 1.0 million (55.56%) were women [30]. While the national HIV prevalence among individuals whose ages fall between 14 and 50 years is 1.4%, women of ages 15 to 49 years are more than two times as probable to be infected with the virus as men (1.9% against 0.9%). The gap in the HIV incidence between men and women is widest among younger individuals, with young females aged 20-24 years old more than thrice as liable to be infected with the virus as young men who are within the same age bracket [31].

In an attempt to provide a cure, drugs can play an important role in limiting transmission. Applied either as a pre-exposure prophylaxis (PrEP) or as a post-exposure prophylaxis (PEP), drugs can prevent infections and limit transmissions. PrEP is recommended for healthy persons who are known to expose themselves to infections while PEP is recommended for those who have been infected in order to halt or reduce transmission. As for malaria, for instance, when a person intends to go to a malaria-endemic area, he is advised to take malaria drugs [10]. The medication might prevent travellers from infections if bitten by infected mosquitoes during their journeys.

The vaccine is arguably the best method of protecting the entire population against infections [37]. Generally, it takes some time for the development and production of vaccines. However, effective vaccines are yet to be developed for a good number of diseases like HIV. The successive attempts by scientists especially in 1994 and 2007 to produce HIV vaccines hit the rock [34]. In the absence of vaccines, drug-oriented strategies can be an alternative intervention for limiting the infection burden [13]. A good number of results have been published confirming the remarkable reduction of HIV viral loads in infected persons due to the application of antiretroviral therapy (ART) [23, 9, 16, 1]. The decrease in viral load is responsible for the reduction in transmission probability. Hence,

a drug-oriented approach could be a potential strategy to alleviate the present burden of HIV infection.

Similarly, considerable success has been recorded on the application of PrEP in limiting HIV infections [22, 32, 15]. PrEP is not meant for everybody: only individuals who have not been infected with the virus but who are at the danger of contracting the virus are recommended to take PrEP [7]. South Africa was the first country in sub-Saharan region of Africa to add PrEP to her national HIV program in 2015 [25]. Many countries have joined South Africa in approving the sale of PrEP. For instance, the market authorization for PrEP has been granted across twenty-eight European Union countries by the European Medicine Agency [36]. PrEP is the application of low-level antiretroviral, such as microbicides to the susceptible individuals, normally, prior to their exposure to possible HIV sources. A microbicide is a substance that is applied inside the vaginal or rectum to significantly minimize the spread HIV and other sexually transmitted infections (STIs). Microbicides can be prepared in many ways: suppositories, creams, gels, films, vaginal rings and slow-releasing sponges [12].

A cohort study [17] based on 889 women with several partners has recorded the immense effect of tenofovir gel, one of the PrEP microbicides, in limiting HIV infections, and its achievement has gained remarkable attention all over the world [8, 21, 20]. Since HIV acquisition is significantly high among women, a good understanding of the efficiency of tenofovir gel might be helpful for adequate implementation of the gel to achieve optimum benefits in checking HIV infections. The gel is used twelve hours of sexual acts. It is applied by inserting it into the vagina within two times in a 24-hour period; one dose, within twelve hours before sex and another dose, within twelve hours after sex [8]. Tenofovir gel is advantageous to women who are at risk as they can apply it without their partners' awareness [17] thereby, preventing any possible objections from the partners. Inspired by the analysis in [23], the present study is aimed at analyzing the effect of tenofovir gel in reducing HIV acquisition in women with reference to Nigeria. The mathematical study of HIV prevention among Nigerian women through the use of vaginal gels is rare in the literature.

A mathematical model of HIV/AIDS transmission with antiretroviral treatment (ARV) and Pre-exposure prophylaxis (PrEP) was formulated and analyzed in [23]. It was discovered that the appropriate use of PrEP and ARV by the individuals who are at the danger

of HIV infections (female sex workers adolescent girls and young women) can limit HIV propagation in the population. However, the researchers in [23] did not base their analysis on a particular PrEP. Besides, the researchers considered five compartments in the analysis and assumed two stages of the infectiousness - asymptomatic phase and symptomatic phase. They assumed that individuals at the symptomatic stage could move back to the asymptomatic phase through treatment. Since both symptomatic and asymptomatic HIV positive individuals are infectious and can spread the virus, the two infectious classes in [23] are lumped together and called infectious class $I(t)$ in the present analysis. Besides, the compartment for AIDS is neglected following [16, 19, 2, 14] since the compartment may not influence the dynamics of the disease. The advancement of infection at the stage will make it difficult for individuals in the AIDS compartment to spread the virus. Therefore, the five compartments in [23] are reduced to three compartments in the present analysis. The present analysis is designed to use a simple susceptible-protected-infectious compartmental model to examine the effect of protection on the minimization of HIV infection among Nigerian women.

2. Model Formulation

A homogeneously mixing population of size $N(t)$ is considered with the population sub-categorized into susceptible class $S(t)$, infective class $I(t)$ (both male and female) and the individuals on tenofovir gel which is protective class, $P(t)$. The flow diagram of the model is in Figure 1.

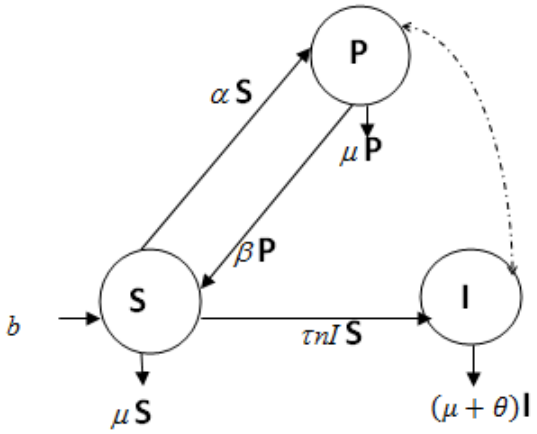

Figure 1: Flow chart of the model

Almost all females are recruited into the S-class at a rate b. Cases of raping of females at all ages are rampant in Nigeria including infants of below 6 months and adults of above 70 years. The crime is so high that the country records 717 rape cases within five months in 2020
(allafrica.com/stories/202006150851.html;
bbc.com/news/world-africa-52994587). Tenofovir gel is introduced to protect the females against HIV acquisition. Those who are under protection move to the protected class P at a rate α . It is assumed that individuals in the protected class do not get an infection even if they have sexual interaction with the individuals in the infectious class. Thus, the dotted arrow in Figure 1 indicates possible interaction which does not result in a transfer. Individuals under tenofovir gel default the prevention and move back to the susceptible compartment at a rate β. The sexual interaction between the susceptible and the infective results in infections with per capita probability τ while n is the average number of sexual partners for an infective. In addition to natural mortality which occurs in each compartment at a rate μ, death due to infection occurs for individuals in the infectious compartment at a rate θ. From the assumptions and Figure 1, the following

equations are obtained.

$$\begin{aligned} \frac{dS}{dt} &= b + \beta P - \alpha S - \tau n I S - \mu S, \\ \frac{dI}{dt} &= \tau n I S - (\mu + \theta) I, \\ \frac{dP}{dt} &= \alpha S - \beta P - \mu P, \end{aligned} \tag{2.1}$$

where:

b : recruitment rate for female population,

β : default rate of protection,

α : rate of protection,

τ : probability of contracting HIV after sexual contact with an infective,

n : average number of sexual partners for an infective,

θ : disease induced death rate,

μ : natural mortality rate.

Following the approach explained in [3], the system (2.1) remains positive and bounded in the region defined by

$$\Gamma = \left\{ (S, I, P) : 0 < S + I + P \leq \frac{b}{\mu} \right\}. \tag{2.2}$$

Besides, the system allows an infection-free equilibrium

$$E^{\circ} = (S^{\circ}, I^{\circ}, P^{\circ}) = \left(\frac{b(\mu + \beta)}{\mu(\mu + \alpha + \beta)}, 0, \frac{\alpha b}{\mu(\mu + \alpha + \beta)} \right). \tag{2.3}$$

In the same vein, the system admits a non infection-free equilibrium $E^* = (S^*, I^*, P^*)$ with each coordinate defined as

$$\begin{aligned} S^* &= \frac{\mu + \theta}{\tau n}, \\ I^* &= \frac{b\tau n(\mu + \beta) - \mu(\mu + \theta)(\mu + \alpha + \beta)}{\tau n(\mu + \theta)(\mu + \beta)}, \\ P^* &= \frac{\alpha(\mu + \theta)}{\tau n(\mu + \beta)}. \end{aligned} \tag{2.4}$$

Also, the reproduction number of the model can be derived from the expression $\frac{1}{S^*} \times S$ [4]. Therefore,

$$R_\circ = \frac{\tau n b(\mu+\beta)}{\mu(\mu+\theta)(\mu+\alpha+\beta)}. \tag{2.5}$$

Non infection-free equilibrium exists only if $R_\circ > 1$. The populations of individuals who are susceptible, infected and protected at this point are given by S^*, I^* and P^*. However, infection does not spread in the women population when $R_\circ < 1$. The population of women that are likely to pick up infection and those who are completely protected from picking infection at this point (i.e., $R_\circ < 1$) are respectively S° and P° while no woman is infected with the virus (i.e. $I^\circ = 0$). Denoting the endemic equilibrium by E^* with cordinates $E^* = (S^*, I^*, P^*)$ then expressing the cordinates of E^* in terms of R_0, E^* becomes

$$E^* = (S^*, I^*, P^*) = \left(\frac{S^\circ}{R_0}, \frac{\mu(\mu+\alpha+\beta)}{\tau n(\mu+\beta)}(R_0 - 1), \frac{\alpha S^\circ}{(\mu+\beta)R_0} \right).$$

Hence, E^* exists only if $R_0 > 1$.

3. Stability Analysis

The local stability of both the infection-free and non infection-free equilibria is to be investigated to examine the behavior of each equilibrium.

Theorem 3.1. *The infection-free equilibrium E° of the model is locally asymptotically stable in Γ if $R_0 < 1$.*

Proof. $R_0 < 1$ if all the eigenvalues of the variational matrix of the system of equations (2.1) about E°, the infection-free equilibrium, are negative. The variational matrix of the system about E° in terms of R_0 is derived as

$$J(E^\circ) = \begin{pmatrix} -(\mu+\alpha) & -\frac{\tau n b(\mu+\beta)}{\mu(\mu+\alpha+\beta)} & \beta \\ 0 & (\mu+\theta)[R_0 - 1] & 0 \\ \alpha & 0 & -(\mu+\beta) \end{pmatrix}.$$

The characteristic equation of $J(E^\circ)$ is

$$((\mu+\theta)[R_0 - 1] - \lambda)(a_2\lambda^2 + a_1\lambda + a_0) = 0,$$

where,

$$a_2 = 1,$$

$$a_1 = \alpha + \beta + 2\mu,$$

$$a_0 = \mu(\alpha + \beta + \mu).$$

Therefore, the roots of $|J(E^\circ)|$ can be obtained from

$$(\mu + \theta)[R_0 - 1] - \lambda = 0,$$

or

$$a_2\lambda^2 + a_1\lambda + a_0 = 0.$$

From $(\mu + \theta)[R_0 - 1] - \lambda = 0$, $\lambda = (\mu + \theta)[R_0 - 1] < 0$ if $R_0 < 1$. Also, since all the parameters are nonnegative then $a_1 > 0$ and $a_0 > 0$ so that all the eigenvalues in $a_2\lambda^2 + a_1\lambda + a_0 = 0$ are negative following Routh-Hurwitz stability criteria [5, 27]. Thus, E° is locally asymptotically stable only if $R_0 < 1$.

It has clearly been shown that the infection-free equilibrium is stable only if $R_0 < 1$. However, the model is built around protection of women against HIV acquisition. Hence, there is a need to draw conclusion in terms of protection default rate β. We shall therefore show the effect of protection default rate on R_0 and the stability of infection-free equilibrium following the same linearization approach.

$$J(E^\circ) = \begin{pmatrix} -(\mu+\alpha) & -\dfrac{\tau nb(\mu+\beta)}{\mu(\mu+\alpha+\beta)} & \beta \\ 0 & \dfrac{\tau nb(\mu+\beta)}{\mu(\mu+\alpha+\beta)} - (\mu+\theta) & 0 \\ \alpha & 0 & -(\mu+\beta) \end{pmatrix}. \tag{3.1}$$

With row reduce matrix operation, (3.1) reduces to

$$J(E^\circ) = \begin{pmatrix} -(\mu+\alpha) & -\dfrac{\tau nb(\mu+\beta)}{\mu(\mu+\alpha+\beta)} & \beta \\ 0 & \dfrac{\tau nb(\mu+\beta)}{\mu(\mu+\alpha+\beta)} - (\mu+\theta) & 0 \\ 0 & -\dfrac{\tau nb(\mu+\beta)}{\mu(\mu+\alpha+\beta)} & -\dfrac{(\mu+\alpha)(\mu+\beta)}{\alpha} + \beta \end{pmatrix}. \tag{3.2}$$

The characteristic equation

$$|J(E^\circ) - \lambda| = 0 \tag{3.3}$$

has the eigenvalues

$$\lambda_1 = -(\mu + \alpha), \tag{3.4}$$

$$\lambda_2 = -\frac{(\mu + \alpha)(\mu + \beta)}{\alpha} + \beta, \tag{3.5}$$

$$\lambda_3 = \frac{\tau nb(\mu + \beta)}{\mu(\mu + \alpha + \beta)} - (\mu + \theta). \tag{3.6}$$

From (3.5), the negativity of λ_2 is a function of default rate of protection β. A quick check shows that λ_2 is negative if β is low. i.e., $\lambda_2 < 0$ as $\beta \to 0$.
From (3.6), $\lambda_3 < 0$ if

$$\frac{\tau nb(\mu + \beta)}{\mu(\mu + \alpha + \beta)} - (\mu + \theta) < 0,$$

$$\Rightarrow \frac{\tau nb(\mu + \beta)}{\mu(\mu + \alpha + \beta)} < (\mu + \theta),$$

$$\Rightarrow \frac{\tau nb(\mu + \beta)}{\mu(\mu + \alpha + \beta)(\mu + \theta)} < 1, \tag{3.7}$$

In view of (2.5), inequality (3.7) becomes

$$R_0 < 1. \tag{3.8}$$

Therefore, $\lambda_3 < 0$ if and only if $R_0 < 1$ and the infection-free equilibrium is locally asymptotically stable otherwise $\lambda_3 > 0$ and E° is locally asymptotically unstable. The biological implication of the result of Theorem 1 is that HIV infection will not spread in the population of Nigerian women if an infected individual gets into the naive population as long as $R_0 < 1$. The feasibility of $R_0 < 1$ is a function of level of adherence to protection. $R_0 < 1$ is likely to be feasible if the rate of default is minimal (i.e. if $\beta \to 0$). □

Theorem 3.2. *The infection-free equilibrium E° is locally asymptotically unstable and consequently, the non infection-free equilibrium E^* becomes locally asymptotically stable in Γ if $R_0 > 1$.*

Proof. $R_0 > 1$ if all the eigenvalues of the variational matrix of the system of equations (2.1) about E^*, the non infection-free equilibrium, are negative. The variational matrix

of the system about E^* is derived as

$$J(S^*, I^*, P^*) = \begin{pmatrix} -\left[A_1 + \dfrac{A_2A_4}{A_3}\right] & -A_5 & \beta \\ \dfrac{A_2A_4}{A_3} & A_5 - A_6 & 0 \\ \alpha & 0 & -A_3 \end{pmatrix}, \tag{3.9}$$

where $A_1 = (\mu + \alpha)$, $A_2 = \mu(\mu + \alpha + \beta)$ $A_3 = (\mu + \beta)$, $A_4 = (R_0 - 1)$, $A_5 = \tau n \dfrac{S^\circ}{R_0}$ and $A_6 = (\mu + \theta)$. With row reduce matrix operation, (3.9) reduces to

$$J(S^*, I^*, P^*) = \begin{pmatrix} -\dfrac{1}{\beta}\left[A_1 + \dfrac{A_2A_4}{A_3}\right] & -\dfrac{A_5}{\beta} & 0 \\ \dfrac{A_2A_4}{A_3} & A_5 - A_6 & 0 \\ \alpha & 0 & -A_3 \end{pmatrix}. \tag{3.10}$$

$\lambda_1 = -A_3$ is one of the eigenvalues of $|J(S^*, I^*, P^*) - \lambda| = 0$. The two other eigenvalues can be obtained from submatrix W given by

$$W = \begin{pmatrix} -\dfrac{1}{\beta}\left[A_1 + \dfrac{A_2A_4}{A_3}\right] & -\dfrac{A_5}{\beta} \\ \dfrac{A_2A_4}{A_3} & A_5 - A_6 \end{pmatrix}. \tag{3.11}$$

The eigenvalues of (3.11) are both negative if $Tr(W) < 0$ and $det(W) > 0$.

$$Tr(W) = -\frac{1}{\beta}\left[A_1 + \frac{A_2A_4}{A_3}\right] - A_6 + A_5. \tag{3.12}$$

$$det(W) = \frac{1}{\beta}(A_6 - A_5)\left[A_1 + \frac{A_2A_4}{A_3}\right] + \frac{1}{\beta}\left[\frac{A_2A_4A_5}{A_3}\right]. \tag{3.13}$$

$R_0 > 1$ and the non infection-free equilibrium is locally asymptotically stable if $Tr(W) < 0$ and $det(W) > 0$ otherwise $R_0 < 1$ and the non infection-free equilibrium is locally asymptotically unstable.

The biological meaning of the result of Theorem 3.2 is that HIV will spread in the population of women if the two conditions if $Tr(W) < 0$ and $det(W) > 0$ are satisfied simultaneously otherwise the virus will fail to spread. The endemic equilibrium of the model might fail to be stable and one of the conditions or both conditions in (3.12) and (3.13) might fail to hold if the rate of protection α and the default rate of protection β are high and low respectively (i.e., $\alpha \to 1$ and $\beta \to 0$). Hence, the two parameters, α and β, play a major role in the spread of HIV in the women population. □

3.1. **Global Stability of the Equilibria.** HIV scourge is a global phenomenon. Local stability analysis of the infection-free and the non infection-free equilibria are not enough to reveal the true picture of the dynamics of a pandemic disease like HIV. Therefore, the analysis of the model shall be extended beyond a small region around the equilibrium by considering the global stability of the infection-free and the non infection-free equilibria.

Theorem 3.3. *The infection-free equilibrium E° of the model is globally asymptotically stable in Γ if $R_0 \leq 1$.*

To investigate the global stability of the infection-free equilibrium, the principle of Lyapunov function is adopted.

Proof. The infection-free equilibrium of the system is globally asymptotically stable in Γ if the time derivative of the Lyapunov function V is less than or equal to zero. The Lyapunov function V is constructed as

$$V(t) = I(t). \tag{3.14}$$

The time derivative of (3.14) is

$$\dot{V} = \dot{I} \tag{3.15}$$

By appropriate substitution, (3.15) becomes

$$\begin{aligned} \dot{V} &= (\tau n S - (\mu + \theta))I, \qquad (3.16)\\ \Rightarrow \dot{V} &= \left(\frac{\tau n b(\mu+\beta)}{\mu(\mu+\alpha+\beta)} - (\mu+\theta)\right) I,\\ \Rightarrow \dot{V} &= (\mu+\theta)\left(\frac{\tau n b(\mu+\beta)}{\mu(\mu+\theta)(\mu+\alpha+\beta)} - 1\right) I,\\ \Rightarrow \dot{V} &= (\mu+\theta)(R_0 - 1)I. \qquad (3.17) \end{aligned}$$

Therefore, $\dot{V} \leq 0$ iff $R_0 \leq 1$ but $\dot{V} = 0$ only if I=0. It implies that as $t \to \infty$, $\{S(t), I(t), P(t)\} \to \left(\frac{b(\mu+\beta)}{\mu(\mu+\alpha+\beta)}, 0, \frac{\alpha b}{\mu(\mu+\alpha+\beta)}\right)$. Hence, $\{E^\circ\}$ is the singleton which remains the largest invariant set in $\{(S(t), I(t), P(t)) \in \Gamma : V = 0\}$. Therefore, following LaSalle's invariance principle in [3], the infection-free equilibrium given by $\{E^\circ\}$ is globally asymptotically stable in Γ if $R_0 \leq 1$. On the other hand, if $R_0 > 1$, $\dot{V} > 0$ and the infection spreads in the women population. The global stability of the non infection-free

equilibrium of the system can therefore be investigated via a suitable nonlinear Lyapunov function. □

Theorem 3.4. *The endemic equilibrium point E^* of the model is globally asymptotically stable in Γ if $R_0 > 1$.*

Proof. The non infection-free equilibrium E^* of the system is globally asymptotically stable in Γ if the time derivative of the nonlinear Lyapunov function L is less than or equal to zero. Define a nonlinear Lyapunov function L as

$$L = \left(S - S^* - S^* \log \frac{S}{S^*}\right) + \left(I - I^* - I^* \log \frac{I}{I^*}\right) + \left(P - P^* - P^* \log \frac{P}{P^*}\right). \tag{3.18}$$

The time derivative of (3.18) is

$$\begin{aligned}
\dot{L} &= \left(\frac{dS}{dt} - \frac{S}{S^*}\frac{dS}{dt}\right) + \left(\frac{dI}{dt} - \frac{I}{I^*}\frac{dI}{dt}\right) + \left(\frac{dP}{dt} - \frac{P}{P^*}\frac{dP}{dt}\right) \\
&= \left(1 - \frac{S}{S^*}\right)\frac{dS}{dt} + \left(1 - \frac{I}{I^*}\right)\frac{dI}{dt} + \left(1 - \frac{P}{P^*}\right)\frac{dP}{dt} \\
&= \left(1 - \frac{S}{S^*}\right)[b + \beta P - \alpha S - \tau n I S - \mu S] \\
&+ \left(1 - \frac{I}{I^*}\right)[\tau n I S - (\mu + \theta) I] \\
&+ \left(1 - \frac{P}{P^*}\right)[\alpha S - \beta P - \mu P] \\
&= \left(\frac{S - S^*}{S^*}\right)[(\mu + \alpha + \tau n I)S^* - (\mu + \alpha + \tau n I)S] \\
&+ \left(\frac{I - I^*}{I^*}\right)[(\mu + \theta)I^* - (\mu + \theta)I] \\
&+ \left(\frac{P - P^*}{P^*}\right)[(\mu + \beta)P^* - (\mu + \beta)P] \\
&= \left(\frac{S - S^*}{S^*}\right)[(\mu + \alpha + \tau n I)](S^* - S) \\
&+ \left(\frac{I - I^*}{I^*}\right)[(\mu + \theta)](I^* - I) \\
&+ \left(\frac{P - P^*}{P^*}\right)[(\mu + \beta)](P^* - P) \\
&= -\left(\frac{S - S^*}{S^*}\right)[(\mu + \alpha + \tau n I)](S - S^*) \\
&- \left(\frac{I - I^*}{I^*}\right)[(\mu + \theta)](I - I^*) \\
&- \left(\frac{P - P^*}{P^*}\right)[(\mu + \beta)](P - P^*)
\end{aligned}$$

$$\Rightarrow \dot{L} = -\left[\frac{(S - S^*)^2}{S^*}(\mu + \alpha + \tau n I) + \frac{(I - I^*)^2}{I^*}(\mu + \theta) + \frac{(P - P^*)^2}{P^*}(\mu + \beta)\right] \tag{3.19}$$

Since $E^* = (S^*, I^*, P^*)$ is a point inside N=(S,I,P) then $S^* \leq S, I^* \leq I$ and $P^* \leq P$. Hence, $\dot{L} \leq 0$. Also, $\dot{L} = 0$ iff $S^* = S$, $I^* = I$ and $P^* = P$. Thereby, by LaSalle's invariant principle in [6], $(S^*, I^*, P^*) \to E^*$ as $t \to \infty$ and E^* is globally asymptotically stable in Γ if $R_0 > 1$ where E^* is the non infection-free equilibrium of the system □

To conduct the simulation in section 4, the values assigned to the parameters are displayed in Table 1.

Table 1. Parameters' values and sources

Parameters	Values	Sources
b	0.187	Estimated
β	0.002	Nominal
α	0.001	Nominal
τ	0.000581	[23]
n	2	[24]
θ	0.0003	Estimated
μ	0.018	Estimated

4. Numerical Simulation and Discussion

The model system can be applied to examine the effectiveness of the intervention against HIV spread and transmission among Nigerian women. Specifically, government and non-governmental organizations may wish to see the impact of, for instance, increasing the application of tenofovir gel in limiting HIV infections among women in Nigeria. Hence, simulations in this regards will be conducted by demonstrating the theoretical results in terms of numerical simulations. The simulations are conducted with the aids of parameter values whose sources are from the literature as well as assumptions. Besides, the values of some of the parameters are estimated within the context of Nigeria.

4.1. **Details on parameter estimations in Table 1.** The population of Nigeria is estimated at 200.96 million in 2019 making the country the seventh most populous in the world. The population of female is lower than that of male standing at 49.4% with 5.349 births per woman. Since the population of male is higher than that of female, the recruitment rate for female population b, is estimated as $\frac{1}{5.349}$ and that of male as

$1 - \frac{1}{5.349}$. Also, in 2019, the average life expectancy for Nigeria is 54.5 years with 53.7 years for men and 55.4 years for women. Generally, the natural death rate is the reverse of the life expectancy, and hence $\mu = \frac{1}{55.4}$ year^{-1}.

The AIDS-related mortality in 2019 is estimated at 53 000 [30]. Since infection rate for women is higher than the infection rate for men in Nigeria, it is assumed that 30 000 women died of AIDS in 2019. The population of women is 49.4% of the total population 200.96 million. Therefore, the AIDS-related mortality rate for women is estimated at $\theta = 30000 \div 49.4\%$ of 200.96 million . The values of other parameters are drawn from the existing works that have bearings with the present HIV situation in Nigeria. For instance, economic hardship has forced many women into extra marital affairs in Nigeria. Thus, the average number of sexual partners, n is taken to be 2. Also, the rate at which women are defaulting protection would be more than the rate at which they are staying on it due to ignorance and poor awareness. Hence, it is assumed that when one out of a thousand women is adhering to protection, two out of a thousand would be defaulting as displayed in Table 1.

4.2. **Initial conditions.** Since 49.4% of 200.96 million individuals are estimated to be women in Nigeria in 2019, the present analysis assumes 75% of the figure (99 274 240) to be susceptible. That is, $S(t_0) = 74\ 455\ 680$. Also, since the study is considering unprotected vaginal sex as a means of contracting HIV for Nigerian women, all HIV positive individuals (both men and women) are considered in the infectious class. The figure for the individuals, according to [30], is 1.8 million. Therefore, $I(t_0) = 1\ 800\ 000$. The population for the infective male is included in the total population for women $N(t_0)$ *i.e.* 99 274 240 because susceptible women can only contract the virus from the infected men. Therefore $P(t_0) = N(t_0) - S(t_0) - I(t_0)$ and $P(t_0) = 23\ 018\ 560$

To begin with the simulation, using the parameter values in Table 1 to evaluate $R_\circ$ in (5), $R_\circ \approxeq 0.6$. Generally, an increase or a decrease in the rate of application of an intervention against a disease will be inversely proportional to the per capita probability of contracting the disease. It is therefore assumed that a ten times increase in the rate of application of the intervention parameter (α) results in a ten times decrease in the per capita probability of contracting the virus parameter (τ). Hence, in Table 1, if $\alpha = 0.01$ then $\tau = 0.0000581$ and $R_\circ \approxeq 0.04$ provided all other parameters are held constant. Also,

if $\alpha = 0.0001$ then $\tau = 0.00581$ and $R_\circ \approxeq 6.6$. The interpretation of the results for $R_\circ$ is that HIV transmission through vaginal sex for Nigerian women could be well inhibited within the framework of the model when $R_\circ \approxeq 0.6$. Better still, the transmission could be driven into extinction if the number of women who stay on protection increased ten times when $R_\circ \approxeq 0.04$. However, infections could escalate and get out of hand if the number of women who default protection increased ten times when $R_\circ \approxeq 6.6$. Therefore, if Nigerian women are to be well protected against HIV acquisition, it is necessary to consider how best to launch tenofovir gel into programs where they can be adequately supported.

The graphical profiles for the dynamics are provided in Figure 2-Figure 4. Figure 2 is achieved by using the parameter values in Table 1 as well as the derived initial conditions for the variables. Figure 3-Figure 4 are plotted by varying the values of the parameters α and τ but keeping the values of other parameters constant as in Table 1.

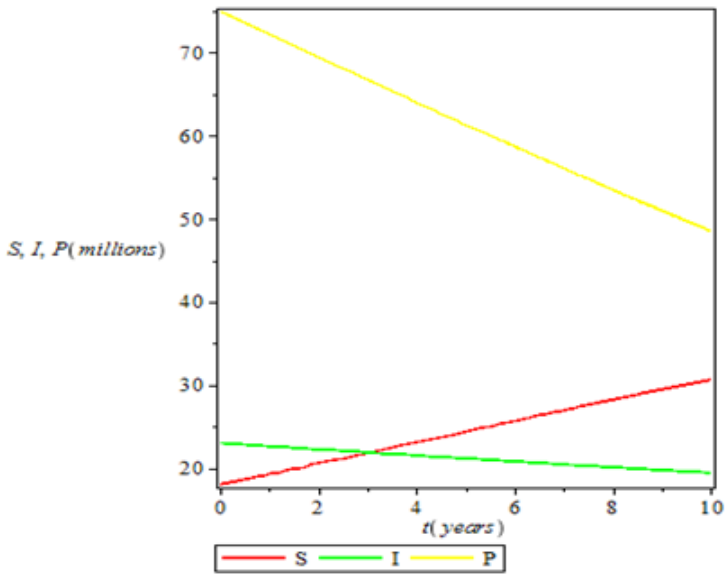

Figure 2. Behavior of the system for $R_0 \cong 0.6$

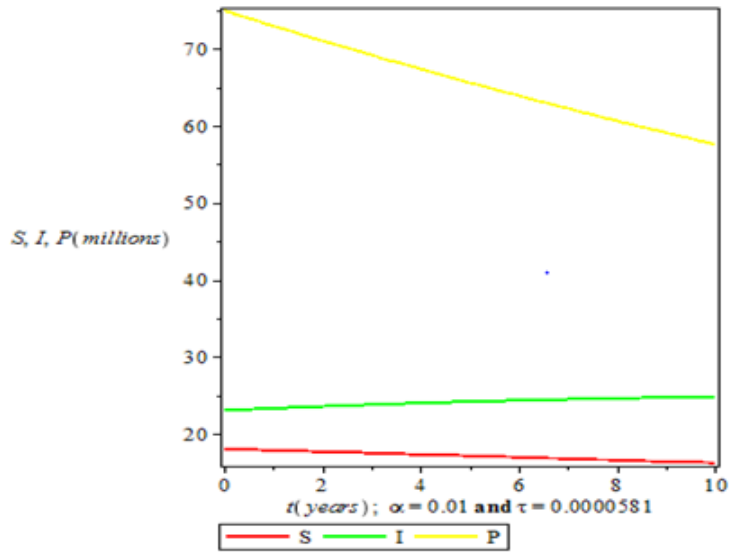

Figure 3. Dynamics of the system for $R_0 \cong 0.04$

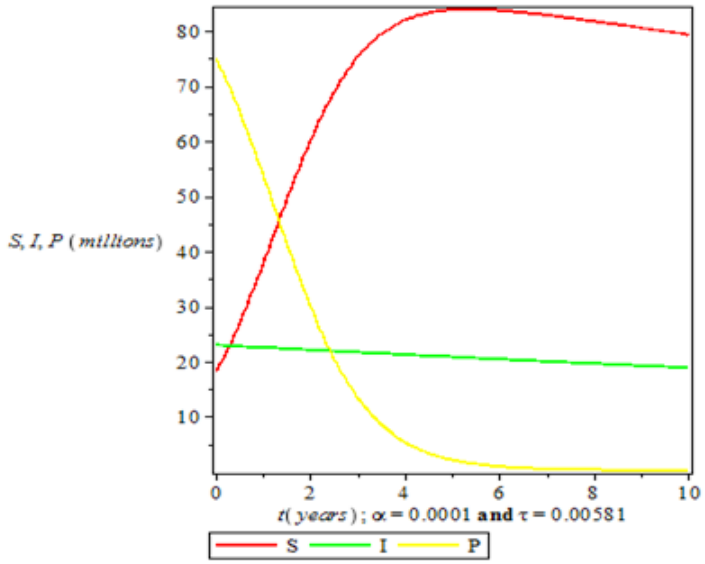

Figure 4. Behavior of the system for $R_0 \cong 6.6$

Figure 2, Figure 3 and Figure 4 display the behavior of the system for $R_\circ \cong 0.6$, $R_\circ \cong 0.04$. and $R_\circ \cong 6.6$ respectively. As regards the conditions for the global stability of the disease-free equilibrium and that of the endemic equilibrium that were established in section 3, the implication of the results is that while the HIV transmission in the population of Nigerian

women is set for an instant failure within the restrictions prescribed by the model when the disease-free equilibrium of the model is globally asymptotically stability, HIV infection is sure to spread in the population of Nigerian women if the endemic equilibrium is globally asymptotically stable.

5. Conclusion

In this work, the methods of protecting Nigerian women against HIV infection has been investigated by considering the application of tenofovir gel. An existing mathematical model is modified and a robust analysis is conducted in terms of the equilibria points and the global stability of the equilibria points. The necessary and sufficient conditions for attaining the global stabilities for both the infection-free and non-infection-free equilibria are established. An epidemic threshold quantity known as the reproduction number is also derived and the numerical analysis is carried out to verify the analytical results. From the simulation, it is discovered that strong adherence to the use of tenofovir gel is an effective means of protecting Nigerian women from HIV infection.

Acknowledgments

We would like to thank the rererees and the editor for valuable comments and helpful suggestions which improve the content and presentation of the paper.

References

[1] Aderinto, Y.O., Iyasere, J.I., Yisa, B.M. and Ejieji, C.N., 2020. Mathematical model for an effective management of HIV infection with a view of window period. *Daffodil International University Journal of Science and Technology, 15*, pp. 21-25.

[2] Aljhani, S., Noorani, M. S. Md. and Alomari, A. K. 2020. Numerical solution of fractional-order HIV model using homotopy method. *Discrete Dynamics in Nature and Society,* https://doi.org/10.1155/2020/2149037

[3] Ayoade, A. A. and Ibrahim, M. O., 2022. Analysis of transmission dynamics and mitigation success of COVID-19 in Nigeria: an insight from a mathematical model. *The Aligarh Bulletin of Mathematics, 41*, pp. 81-106.

[4] Ayoade, A. A., Agboola, S. O. and Ibrahim, M. O., 2019. Mathematical analysis of the effect of maternal immunity on the global eradication of measles. *Anale.Seria Informatica, 17*, pp. 235-241.

[5] Ayoaade, A. A. and Thota, S., 2023. Functional education as a nexus between agricultural and industrial revolution: an epidemiological modeling approach. *Uniciencia, 37*, pp. 1-16.

[6] Ayoade, A. A., Oyedepo, T. and Agunbiade, S., 2023. Mathematical modeling of Toxoplasma gondii between the environment and cat population under vaccination and sanitation. *Journal of Fractional Calculus and Applications(JFCA), 14*, pp. 75-87.

[7] Babayemi, O.O., Daniel, S. W., Sabastine, S.W., Tolulope, T.O. and Chamberline, E. O., 2018. Quantification of the effect of terrorism on the HIV response in Nigeria. *The Lancet*, *391*, pp. 1257-1260.

[8] BBC New, 2017. Scientists say vaginal gel cuts HIV-infections by half. Retrieved February 7, 2022 from http://www.bbc.co.uk/news/health-10691353

[9] Campos, C., Silva, C. J. and Torres, D. F. M., 2019. Numerical optimal control of HIV transmission in octave/matlab. *Math. Comput. Appl., 25*, pp. 1-20. doi:10.3390/mca25010001

[10] CDC, 2020. Travelling? Make sure you protect yourself from malaria, Retrieved January 4, 2022 from http://www.cdc.gov/malaria/resources/pdf/travelers.pdf

[11] Etiaba, E., Onwujekwe, O., Torpey, K., Uzochukwu, B. and Chiegil, R. 2016. What is the economic burden of subsidized HIV/AIDS treatment services on patients in Nigeria and is this burden catastrophic to households?. *Plos One*, *11*, e0167117.

[12] European Medicine Agency, 2015. First medicine for HIV pre-exposure prophylaxis recommended for approval in the EU. Retrieved September 5, 2022.

[13] Fagbamigbe, A. F., Adebayo, S. B. and Idemudia, E., 2016. Marital status and HIV prevalence among women in Nigeria: ingredients for evidence-based programming. *International Journal of Infectious Diseases, 48*, pp. 57-63.

[14] Ferrari, A. J., Lara, L. P., Olguin, M. C. and Santillan- Marcus, E. A., 2022. Comparative study of some numerical schemesfor a fractional order model of HIV infection treatment. *Trends in Computational and Applied Mathematics, 23*, pp. 607-623.

[15] Hernandez-Vargas, B. J. and Middleton, R. H., 2013. Modeling the three stages of HIV infection. *Journal of Theoretical Biology, 320*, pp. 33-40.

[16] Jawaz, M., Ahmed, N., Baleanu, D., Rafiq, M. and Rehman, M. A, 2020. Positivity preserving technique for the solution of HIV/AIDS reaction diffusion model with time delay. *Frontier in Physics, 7*, pp. 1-10.

[17] Karim, Q. A., Karim, S. S. A., Frohlich, J. A., Grobler, A. C., Baxter, C., Mansoor, L. E. *et al.,* 2017. Effectiveness and safety of tenofovir gel, an antiretroviral microbicide, for the prevention of HIV infection in women, *Science, 329*, pp. 1168-1174.

[18] Kerrigan, D. and Wirtz, A., 2016. The global HIV epidemics among sex workers. *The World Bank publications.*

[19] Kimber, A. R. and Oduwole, H. K., 2008. A mathematical model of HIV/AIDS transmission dynamics considering counseling and antiretroviral therapy, *Journal of Modern Mathematics and Statistics, 2*, pp. 166-169.

[20] Mesquita, P. M. M., Rastogi, R. , Segarra, T.J., Tella, G., Torres, R.S., Kiser, N.M. *et al.,* 2016. Intra virginal ring delivery of tenofovirdisoproxi; fumerate for prevention of HIV and herpes simplex virus infection. *J. Antimicrob. Chemother., 67*, pp. 1730-1738.

[21] M. M. News, 2017. Tenofovir vaginal gel first microbicide to prevent HIV, HSV infections. Retrieved June 20, 2022 from http://www.medscape.com/viewarticle/725583

[22] NAtional Agency for Control of AIDS, 2017. National Strategic Framework 2017-2012, [pdf].

[23] Nsuami, M. U. and Witbooi, P., 2018. A model of HIV/AIDS population dynamics including ARV treatment and pre-exposure prophylaxis. *Advances in Difference Equations, 11*, pp. 1-12. https://doi.org/10.1186/s13662-017-1458-x

[24] Nyabadza, F. and Mukandavire, Z., 2011. Modelling HIV/AIDS in the presence of an HIV testing and screening campaign, *Journal of Theoretical Biology, 280*, pp. 167-179.

[25] Okosun, K. O., Makinde, O. D. and Takaida, I., 2013. Analysis of recruitment and industrial human resources management for optimal productivity in the presence of HIV/AIDS epidemic. *Journal of Biological Physics, 39*, pp. 99-121.

[26] The Punch Nigeria (September 20, 2019). 10 000 living with HIV in Imo. Available online at punchng.com/10000-living-with-hiv-in-imo/

[27] Thota, S., Gemechu, T. and Ayoade, A. A., 2023. On new hybrid root-finding algorithms for solving transcendental equations using exponential and Halley's method, *Ural Mathematical Journal, 9*(1), pp. 176-186. http://dx.doi.org/10.15826/umj.2023.1.016

[28] UNAIDS, 2018. Number of people living with HIV. Retrieved on May 1, 2022 from http://aidsinfo.unaids.org/

[29] UNAIDS, 2017. West and Central Africa catch up plan. [pdf]

[30] UNAIDS, 2019. HIV/AIDS epidemics in Nigeria. Retrieved on June 4, 2019 from https://www.unaids.org/en/regionscountries/countries/nigeria

[31] UNAIDS, 2019. Current success in HIV/AIDS fight in Nigeria.Retrieved on January 12, 2022 from https://www.unaids.org/en/resources/presscentre/pressreleaseandstatementarchive/ 2019/march/20190314nigeria

[32] UNAIDS, 2016. Prevention gap report. [pdf]

[33] UNAIDS, 2018. Start free, Stay free, AIDS free: 2017 progress report. [pdf]

[34] UNAID Statement, 2007. Immunizations are discontinued in two HIV vaccines trials. Retrieved December 5, 2018 from http://www.naid.nih.gov/news/newsreleases/2007/pages/stepstatement.aspx

[35] WHO, 2018. WHO report on the global HIV trend. Retrieved on February 5, 2022 from: https://www.who.1nt/factsheet/detail/

[36] WHO, 2016. Microbicide. Available at: https://www.who.int/hiv/topics/microbicides/en/

[37] Williamson, T. R. and Rodd, J., 2016. Civil society advocacy in Nigeria: promoting democratic norms and donor demand?, *BMC Int. Health Hum. Rights, 16*, pp. 7-15.

Abayomi Ayotunde Ayoade
Department of Mathematics,
Faculty of Science,
University of Lagos,
Lagos, Nigeria
Email: ayoadeabbayomi@gmail.com; ayoayoade@unilag.edu.ng

Samuel Tosin Akinyemi
Department of Mathematics,
College of Science,
Sikiru Adetona College of Education,
Science and Technology, Omu-Ijebu, Nigeria
Email: sammysalt@gmail.com

Taiye Oyedepo
Department of Applied Sciences,
Faculty of Pure and Applied Sciences,
Federal College of Dental Technology and Therapy,
Enugu, Nigeria
Email: oyedepotaiye@yahoo.com