Journal of Advanced Mathematical Modeling
ISSN (print): 2251-8088, ISSN (on-line): 2645-6141
Vol. 14 No. 3 (2024), pp. 121-129.
DOI: x 10.22055/jamm.2025.47571.2298

SOME PROPERTIES OF THE GRAPH OF MODULES WITH RESPECT TO A FIRST DUAL HOMOMORPHISM

M. BAZIAR*, S. ZARE KHAFRI

Communicated by: Mehrdad Namdari

ABSTRACT. For an R-module M and $f \in M^* = \mathrm{Hom}(M,R)$, let $Z^f(M)$ and $\mathrm{Reg}^f(M)$ be the sets of all zero-divisors elements and regular elements of M with respect to f, respectively. In this paper, we introduce the total graph of M with respect to f, denoted by $T(\Gamma^f(M))$, which is the graph with all the elements of M as vertices, and for distinct elements $m,n \in M$, m and n are adjacent if and only if $m+n \in Z^f(M)$. We also study the subgraphs $Z(\Gamma^f(M))$ and $\mathrm{Reg}(\Gamma^f(M))$ with vertices $Z^f(M)$ and $\mathrm{Reg}^f(M)$, respectively.

1. Introduction

In this paper, every ring is a commutative ring with identity and every module is unitary. The main idea of the zero-divisor graph of a ring R was first introduced by Beck [9]. He takes all the elements of R and $x, y \in R$ are adjacent if and only if $xy = 0$. In [3], Anderson and Badawi introduced the total graph of R, denoted by $T(\Gamma(R))$, taking all elements of R as vertices, the vertices x and y are adjacent if and only if $x + y \in Z(R)$. $\mathrm{Reg}(\Gamma(R))$ is the subgraph of $T(\Gamma(R))$ with vertices $\mathrm{Reg}(R) = R \backslash Z(R)$ and $x, y \in \mathrm{Reg}(R)$ are adjacent if and only if $x+y \in Z(R)$. In [4] and [5], Anderson and Badawi continue to research on total graphs. For further results on total graphs on algebraic structures see

MSC(2020): 05C25, 05C38, 16D10.
Keywords: Total graph, zero-divisor, regular element.
Received: 28 July 2024, Accepted: 17 April 2025.
*Corresponding author.

[1], [2], [6], [10]-[18].

Let M be an R-module and $f \in M^* = \mathrm{Hom}(M,R)$ be an R-homomorphism. We denote the zero-divisor elements of M with respect to f by $Z^f(M)$ where

$$Z^f(M) = \{m \in M \mid mf(n) = 0 \text{ or } nf(m) = 0 \text{ for some } 0 \neq n \in M\}$$

then the zero-divisor graph of M with respect to f is denoted by $\Gamma^f(M)$. For distinct elements $x, y \in Z^f(M)$, x and y are adjacent if and only if $xf(y) = 0$ or $yf(x) = 0$. The zero-divisor graph of M with respect to f has been studied extensively in [7] and [8]. In the next section we consider the graph of an R-module M with respect to $f \in \mathrm{Hom}\ (M,R)$.

Let G be an (undirected) graph. We say that G is *connected* if there exists a path between any two distinct vertices. For distinct vertices x and y in G, the *distance* between x and y, denoted by $d(x,y)$, is the length of a shortest path connecting x and y ($d(x,x) = 0$ and $d(x,y) = \infty$ if no such path exists). The *diameter* of G is

$$\mathrm{diam}(G) = \sup\{d(x,y) \mid \ x \text{ and } y \text{ are vertices of } G\}.$$

A *cycle* of length n in G is a path of the form $x_1 - x_2 - x_3 - \cdots - x_n - x_1$, where $x_i \neq x_j$ when $i \neq j$. We define the *girth* of G, denoted by $\mathrm{gr}(G)$, as the length of a shortest cycle in G, provided G contains a cycle; otherwise, $\mathrm{gr}(G) = \infty$. A graph is *complete* if any two distinct vertices are adjacent. By a complete subgraph we mean a subgraph that is complete as a graph. In this article all subgraphs are induced subgraphs, where a subgraph G' of a graph G is an *induced subgraph* of G if two vertices of G' are adjacent in G' if and only if they are adjacent in G. The reader is referred to [19] and [20] for undefined terms and concepts.

2. When $Z^f(M)$ is a submodule

Let M be an R-module and $f \in M^* = \mathrm{Hom}(M,R)$ be an R-homomorphism. We denote the zero-divisor elements of M with respect to f by $Z^f(M)$ and the set of non-zero-divisor (regular) elements of M with respect to f by $\mathrm{Reg}^f(M) = M - Z^f(M)$. The total graph of M with respect to f, denoted by $T(\Gamma^f(M))$, is a graph with vertices

$$V = \{m \in M \mid m + n \in Z^f(M) \text{ for some } n \in M\}$$

and two elements m, n in V are adjacent if and only if $m + n \in Z^f(M)$.

Let M be an R-module, the proper submodule N of M is called a *prime submodule* if $mr \in N$, for $r \in R$ and $m \in M$ implies that $m \in N$ or $Mr \subseteq N$. By the definition of $Z^f(M)$ it is clear that in general $Z^f(M)$ is not a submodule of M. The next proposition state that $Z^f(M)$ as a submodule is prime.

Proposition 2.1. *Let R be a ring, M be an R-module and $f \in M^*$. If $Z^f(M)$ is a proper submodule of M, then it is a prime submodule of M.*

Proof. Let $mr \in Z^f(M)$ and $m \notin Z^f(M)$. Then, there exists $0 \neq y \in M$ such that $mrf(y) = 0$ or $yf(mr) = 0$. In any case, we must have $yr = 0$. Therefore, $Mrf(y) = 0$ and consequently $Mr \subseteq Z^f(M)$ which implies that $r \in (M : Z^f(M))$. □

Corollary 2.2. *Let M be an R-module and $f \in M^*$ such that $Z^f(M)$ is a submodule of M, then $2x \in Z^f(M)$ for all $x \in$Reg(M) if and only if for all $y \in M$, $2y \in Z^f(M)$.*

Proof. If $2x \in Z^f(M)$, by Proposition 2.1, since $Z^f(M)$ is a submodule of M, so it is a prime submodule of M. Also $x \notin Z^f(M)$, then $2M \subseteq Z^f(M)$. The converse is clear. □

Let R be a ring, M be an R-module and $0 \neq f \in M^*$. One may inquire about the relation between the zero-divisor graph of R and the zero-divisor graph of M with respect to f. The following proposition and theorem state some relations between $Z(R)$ and $Z^f(M)$.

Proposition 2.3. *Let M be an R-module and f be an epimorphism in M^*. Then $2 \in Z(R)$ if and only if $2x \in Z^f(M)$ for all $x \in$Reg$^f(M)$.*

Proof. Let $2 \in Z(R)$, then there exists $0 \neq r \in R$ such that $2r = 0$. So there exists $y \in M$ such that $r = f(y)$. Therefore $2xf(y) = 0$, hence $2x \in Z^f(M)$.
Conversely, if $2x \in Z^f(M)$ for some $x \in$Reg$^f(M)$, then we have two cases. In the first case, there exists $y \in M$ such that $2xf(y) = xf(2y) = 0$, by regularity of x we conclude that $f(2y) = 0$. Hence $2f(y) = f(2y) = 0$ which implies that $2 \in Z(R)$.

In the second case, we have $yf(2x) = 0$, then $2yf(x) = 0$, so $2y = 0$, $f(2y) = 2f(y) = 0$. Now, if $f(y) \neq 0$ then $2 \in Z(R)$, and if $f(y) = 0$, then $xf(y) = 0$, which is a contradiction. □

Proposition 2.4. *Let M be an R-module and $f \in M^*$ be an epimorphism. If the identity of the ring R is a sum of n zero-divisors, then every element of the module M is the sum of at most n elements of $Z^f(M)$.*

Proof. If $a \in Z(R)$ and $x \in M$, then there exists $0 \neq r \in R$ such that $ar = 0$, so there exists $0 \neq t \in M$ such that $r = f(t)$ and $axf(t) = 0$ and hence $ax \in Z^f(M)$. So for all $m \in M$

$$1 = z_1 + ... + z_n, \text{ for some } z_i \in Z(R) \Longrightarrow m = z_1 m + ... + z_n m$$

is the sum of at most n element of $Z^f(M)$. $\square$

Corollary 2.5. *Let the identity of a ring R be a sum of n zero-divisors, M be an R-module and $f \in M^*$ be an epimorphism. Then $Z^f(M)$ is submodule of M if and only if $Z^f(M) = M$.*

Example 2.6. Let $R = \mathbb{Z}_6$ and $M = \mathbb{Z}_6$ as a $\mathbb{Z}_6$-module. We know that $Z(R) = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}\}$. For $f : M \to R$ via $f(\bar{x}) = 2\bar{x}$ we have $Z^f(M) = \mathbb{Z}_6$. Figure 1 shows the total zero-divisor graph of $\mathbb{Z}_6$ as a ring and Figure 2 shows total zero-divisor graph of $\mathbb{Z}_6$ as a module over itself with respect to f (this shows that the condition f be an epimorphism in Corollary 2.5 cannot be omitted).

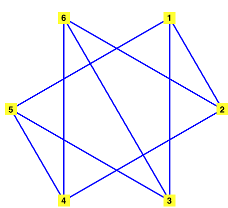

FIGURE 1

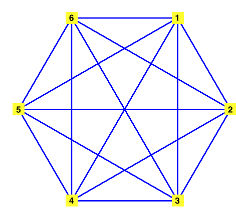

FIGURE 2

The next theorem gives a partial answer to this question, when $Z(R)$ being an ideal implies that $Z^f(M)$ is submodule of M?

Theorem 2.7. *If $Z(R) = \langle z \rangle$ is a principal ideal of R and $z \in Nil(R)$, then $Z^f(M)$ is the submodule of M.*

Proof. Let $Z(R) = \langle z \rangle$ such that $z \in Nil(R)$ and assume that $Z^f(M)$ is not a submodule of M, then there exist $m_1, m_2 \in Z^f(M)$ such that $m_1 + m_2 \notin Z^f(M)$.
So there exist $n_1, n_2 \in Me$ such that

1) $m_1 f(n_1) = 0$ or $n_1 f(m_1) = 0$

2) $m_2f(n_2) = 0$ or $n_2f(m_2) = 0$

a) If $m_1f(n_1) = 0$ and $m_2f(n_2) = 0$, then $(m_1+m_2)f(n_1f(n_2)) = 0$, thus $n_1f(n_2) = 0$ (because $m_1 + m_2 \notin Z^f(M)$), $f(n_1)f(n_2) = 0$ and so $f(n_1), f(n_2) \in Z(R) = \langle z\rangle$. Therefore $f(n_1) = az^k$ and $f(n_2) = bz^t$ such that $a, b \notin Z(R)$. Let $k \geq t$, then $(m_1 + m_2)bf(n_1) = 0$ ($bf(n_1) \neq 0$), so $m_1 + m_2 \in Z^f(M)$ which is contrary to assumption that $m_1 + m_2 \notin Z^f(M)$.

b) If $n_2f(m_2) = 0$ and $m_1f(n_1) = 0$, then $f(m_1)f(n_1) = 0$, $n_2f(n_1)f(m_1 + m_2) = 0$ and so $n_2f(n_1) = 0$ (because $m_1 + m_2 \notin Z^f(M)$), $f(n_1)f(n_2) = 0$, therefore $f(n_1), f(n_2) \in Z(R) = \langle z\rangle$.
So $f(n_1) = az^k$ and $f(n_2) = bz^t$ such that $a, b \notin Z(R)$. Let $k \geq t$, then $bf(n_1)f(m_1 + m_2) = 0$ so $f(n_1)f(m_1 + m_2) = 0$. Now, we consider two cases:

1) If $Mf(n_1) = 0$, then $(m_1 + m_2)f(n_1) = 0$ which is a contradiction.
2) If $Mf(n_1) \neq 0$, then there exists $0 \neq x \in M$ such that $xf(n_1) \neq 0$ and $xf(n_1)f(m_1 + m_2) = 0$ so it is contrary.

c) If $n_2f(m_2) = 0$ and $n_1f(m_1) = 0$, then $f(n_2)f(m_2) = 0$, so $n_1f(n_2)f(m_1+m_2) = 0$, $n_1f(n_2) = 0$ (because $m_1 + m_2 \notin Z^f(M)$), thus $f(n_1)f(n_2) = 0$ and it is similar to (b).

Thus $Z^f(M)$ is a submodule of M. □

Let M be an R-module and $Z^f(M)$ be a submodule of M. Then $Z(\Gamma^f(M)$ (the induced subgraph of $T(\Gamma^f(M))$ by $Z^f(M)$) is a complete subgraph of $T(\Gamma^f(M))$. Since for every $x, y \in Z^f(M)$ we have $x + y \in Z^f(M)$ and so x —y is a path in $Z(\Gamma^f(M))$. It is also worth mentioning that $Z(\Gamma^f(M))$ is disjoint from the induced subgraph of $T(\Gamma^f(M))$ by $\mathrm{Reg}^f(M)$ which is denoted by $\mathrm{Reg}(\Gamma^f(M))$, since for $x \in Z^f(M)$ and $y \in \mathrm{Reg}^f(M)$ we have $x + y \in \mathrm{Reg}^f(M)$ and so $Z(\Gamma^f(M))$ and $\mathrm{Reg}(\Gamma^f(M))$ are disjoint subgraphs of $T(\Gamma^f(M))$.

The next proposition determines regular subgraph of total graph under some conditions.

Theorem 2.8. *Let M be an R-module, $Z^f(M)$ be a submodule of M, $f \in M^*$ be an epimorphism, $|Z^f(M)| = \alpha$ and $|\frac{M}{Z^f(M)}| = \beta$.*

1) *If* $2 \in Z(R)$*, then Reg*$(\Gamma^f(M))$ *is the union of* $\beta - 1$ *disjoint* K^{α}*'s.*
2) *If* $2 \notin Z(R)$*, then Reg*$(\Gamma^f(M))$ *is the union of* $\frac{\beta-1}{2}$ *disjoint* $K^{\alpha,\alpha}$*'s.*

Proof. 1) Assume that $2 \in Z(R)$ and let $x \in \mathrm{Reg}^f(M)$. Then each coset $x + Z^f(M)$ is a complete subgraph of $\mathrm{Reg}(\Gamma^f(M))$ since by Proposition 2.3 $(x + z_1) + (x + z_2) = 2x+z_1+z_2 \in Z^f(M)$ for all $z_1, z_2 \in Z^f(M)$. Distinct cosets form disjoint subgraphs of $\mathrm{Reg}(\Gamma^f(M))$ since if $x + z_1$ and $y + z_2$ are adjacent for some $x, y \in \mathrm{Reg}^f(M)$ and $z_1, z_2 \in Z^f(M)$, then $x + y = (x + z_1) + (y + z_2) - (z_1 + z_2) \in Z^f(M)$, and hence by Proposition 2.3 $x - y = (x + y) - 2y \in Z^f(M)$ since $Z^f(M)$ is a submodule of M and $2 \in Z(R)$, which implies that $x + Z^f(M) = y + Z^f(M)$. It is a contradiction. Thus $\mathrm{Reg}(\Gamma^f(M))$ is the union of $\beta - 1$ disjoint subgraphs $x + Z^f(M)$, each of which is a K^{α}.

2) Now assume that $2 \notin Z(R)$, and let $x \in \mathrm{Reg}^f(M)$. Then no two distinct elements in $x + Z^f(M)$ are adjacent since $(x + z_1) + (x + z_2) \in Z^f(M)$ for $z_1, z_2 \in Z^f(M)$ implies that $2x \in Z^f(M)$ and so $2 \in Z(R)$ which is contradiction. Also, two cosets $x + Z^f(M)$ and $-x + Z^f(M)$ are disjoint, and each element of $x + Z^f(M)$ is adjacent to each element of $-x + Z^f(M)$. Thus $(x + Z^f(M)) \cup (-x + Z^f(M))$ is a complete bipartite subgraph of $\mathrm{Reg}(\Gamma^f(M))$. Furthermore, if $y + z_1$ is adjacent to $x + z_2$ for some $x, y \in \mathrm{Reg}^f(M)$ and $z_1, z_2 \in Z^f(M)$, then $x + y \in Z^f(M)$ and hence $y + Z^f(M) = -x + Z^f(M)$. Thus $\mathrm{Reg}(\Gamma^f(M))$ is the union of $(\beta - 1)/2$ disjoint subgraphs $(x + Z^f(M)) \cup (-x + Z^f(M))$, each of which is a $K^{\alpha,\alpha}$.

□

The set of zero-divisors $Z^f(M)$ is a submodule of M if and only if $\langle Z^f(M)\rangle$ (the induced subgraph of $T(\Gamma^f(M))$ by $Z^f(M)$) is a complete connected component of $T(\Gamma^f(M))$. In the next theorem we characterized when a regular subgraph is complete, connected and totally disconnected.

Theorem 2.9. *Let* M *be an* R*-module and* $f \in M^*$ *an epimorphism. If* $Z^f(M)$ *is a submodule of* M*, then*

1) *Reg*$(\Gamma^f(M))$ *is complete if and only if either* $|\frac{M}{Z^f(M)}| = 2$ *or* $|\frac{M}{Z^f(M)}| = |M| = 3$.
2) *Reg*$(\Gamma^f(M))$ *is connected if and only if either* $|\frac{M}{Z^f(M)}| = 2$ *or* $|\frac{M}{Z^f(M)}| = 3$.

Proof. Let $|\frac{M}{Z^f(M)}| = \beta$ and $|Z^f(M)| = \alpha$.

1) Let $\mathrm{Reg}(\Gamma^f(M))$ be complete, then by Theorem 2.8, $\mathrm{Reg}(\Gamma^f(M))$ is a single K^α or $K^{1,1}$. If $2 \in Z(R)$, then $\beta - 1 = 1$ and so $\beta = 2$, hence $|\frac{M}{Z^f(M)}| = 2$. If $2 \notin Z(R)$, then $\alpha = 1$ and $\frac{(\beta-1)}{2} = 1$. Thus $\beta = 3$ and $Z^f(M) = \{0\}$, hence $|M| = |\frac{M}{Z^f(M)}| = 3$.

   Conversely, suppose that $\frac{M}{Z^f(M)} = \{Z^f(M), x+Z^f(M)\}$, where $x \notin Z^f(M)$, then with this fact $x + Z^f(M)$ and $-x + Z^f(M)$ are adjacent, we have $x + Z^f(M) = -x + Z^f(M)$ and so $2x \in Z^f(M)$, thus $2 \in Z(R)$. Therefore $\mathrm{Reg}(\Gamma^f(M))$ is the union of $\beta - 1$ subgraph K^α of $T(\Gamma^f(M))$. Since $\beta = 2$, so it is a single graph K^α.

   Now suppose that $|\frac{M}{Z^f(M)}| = |M| = 3$. We show that $2 \notin Z(R)$. If $2 \in Z(R)$, since f is an epimorphism, thus $2m \in Z^f(M) = \{0\}$ and $2m = 0$ which is a contradiction, since M is a cyclic group with order 3. Thus $2 \notin Z(R)$ and so $\beta = 3$, thus $\frac{(\beta-1)}{2}$. Therefore $\mathrm{Reg}(\Gamma^f(M))$ is complete.

2) Let $\mathrm{Reg}(\Gamma^f(M))$ is connected, then by 2.8, $\mathrm{Reg}(\Gamma^f(M))$ is a single K^α or $K^{\alpha,\alpha}$. Then either $\beta - 1 = 1$ if $2 \in Z(R)$ or $\frac{(\beta-1)}{2} = 1$ if $2 \notin Z(R)$; hence $\beta = 2$ or $\beta = 3$ respectively. Thus $|\frac{M}{Z^f(M)}| = 2$ or $|\frac{M}{Z^f(M)}| = 3$.

   conversely by part (1) above, if $|\frac{M}{Z^f(M)}| = 2$, then $\mathrm{Reg}(\Gamma^f(M))$ is connected and if $|\frac{M}{Z^f(M)}| = 3$, then we show that $2 \notin Z(R)$. Suppose that $2 \in Z(R)$ and $\frac{M}{Z^f(M)} = \{Z^f(M), x + Z^f(M), y + Z^f(M)\}$, where $x, y \notin Z^f(M)$. Since $\frac{M}{Z^f(M)}$ is a cyclic group with order 3, we conclude that $x+y+Z^f(M) = Z^f(M)$. Hence x and y is adjacent, a contradiction since $\mathrm{Reg}(\Gamma^f(M))$ is the union of 2 disjoint (induced) subgraphs $x + Z^f(M)$ and $y + Z^f(M)$. Thus $2 \notin Z(R)$. By hypothesis, $\frac{M}{Z^f(M)} = \{Z^f(M), x + Z^f(M), 2x + Z^f(M)\}$, where $x, 2x \notin Z^f(M)$ and $3x \in Z^f(M)$.

   Let $m, m' \in \mathrm{Reg}^f(M)$. Without loss of generality, we may assume that $x + Z^f(M) \neq m + Z^f(M)$ and $m + m' \notin Z^f(M)$. Then $2x + Z^f(M) = m + Z^f(M)$. If $x + Z^f(M) = m' + Z^f(M)$, then $m + m' + Z^f(M) = 3x + Z^f(M) = Z^f(M)$ which is contradiction with this fact $m + m' \notin Z^f(M)$. So we may assume that $2x + Z^f(M) = m' + Z^f(M)$, thus $m - (m + m' - 6x) - m'$ is a path in $\mathrm{Reg}(\Gamma^f(M))$ since:

$$(2m - 4x) + (m' - 2x) \in Z^f(M)$$

$$(m - 2x) + (2m' - 4x) \in Z^f(M)$$

and so $\mathrm{Reg}(\Gamma^f(M))$ is connected.

$\square$

Acknowledgments

I would also like to express my gratitude to the meticulous referees for their careful reading, constructive comments and corrections.

References

[1] Abbasi, A. and Habibi, S., 2012. The total graph of a commutative ring with respect to proper ideals. *Journal of the Korean Mathematical Society*, 49(1), pp.85-98. doi:10.4134/JKMS.2012.49.1.085

[2] Akbari, S., Kiani, D., Mohammadi, F. and Moradi, S., 2009. The total graph and regular graph of a commutative ring. *Journal of pure and applied algebra*, 213(12), pp.2224-2228. doi:10.1016/j.jpaa.2009.03.013

[3] Anderson, D.F. and Badawi, A., 2008. The total graph of a commutative ring. *Journal of algebra*, 320(7), pp.2706-2719. doi:10.1016/j.jalgebra.2008.06.028

[4] Anderson, D.F. and Badawi, A., 2012. On the total graph of a commutative ring without the zero element. *Journal of Algebra and Its Applications*, 11(04), p.1250074. doi:10.1142/S0219498812500740

[5] Anderson, D.F. and Badawi, A., 2013. The generalized total graph of a commutative ring. *Journal of algebra and its applications*, 12(05), p.1250212. doi:10.1142/S021949881250212X

[6] Asir, T. and Chelvam, T.T., 2013. The intersection graph of gamma sets in the total graph of a commutative ring-II. *Journal of Algebra and Its Applications*, 12(04), p.1250199. doi:10.1142/S021949881250199X

[7] Baziar, M., Momtahan, E. and Safaeeyan, S., 2013. A zero-divisor graph for modules with respect to their (first) dual. *Journal of Algebra and its applications*, 12(02), p.1250151. doi:10.1142/S0219498812501514

[8] Momtahan, E., Baziar, M. and Safaeeyan, S., 2016. A module theoretic approach to zero-divisor graph with respect to (first) dual *Bulletin of the Iranian Mathematical Society*, 42(4), pp.861-872. http://bims.iranjournals.ir

[9] Beck, I., 1988. Coloring of commutative rings. *Journal of algebra*, 116(1), pp.208-226. doi:10.1016/0021-8693(88)90202-5

[10] Atani, S.E. and Habibi, S., 2011. The total torsion element graph of a module over a commutative ring. *An. St. Univ. Ovidius Constanta*, 19(1), pp.23-34. https://eudml.org/doc/224500

[11] Maimani, H.R., Wickham, C. and Yassemi, S., 2012. Rings whose total graphs have genus at most one. *The Rocky Mountain Journal of Mathematics*, 42(5), pp.1551-1560. https://www.jstor.org/stable/26758214

[12] Pucanović, Z., 2011. The total graph of a module. *Matematički vesnik*, 63(246), pp.305-312. https://euclid.matf.bg.ac.rs/index.php/vesnik/article/view/mv11408

[13] Pucanović, Z. and Petrović, Z.Z., 2011. On the radius and the relation between the total graph of a commutative ring and its extensions. *Publications de l'Institut Mathematique*, 89(103), pp.1-9. https://eudml.org/doc/256291

[14] Ramin, A., 2013. The total graph of a finite commutative ring. *Turkish Journal of Mathematics*, 37(3), pp.391-397. doi:10.3906/mat-1101-70

[15] Shekarriz, M.H., Shirdareh Haghighi, M.H. and Sharif, H., 2012. On the total graph of a finite commutative ring. *Communications in algebra*, 40(8), pp.2798-2807. doi:10.1080/00927872.2011.585680

[16] Chelvam, T.T. and Asir, T., 2013. Domination in the total graph of a commutative ring. *J. Combin. Math. Combin. Comput*, 87, pp.147-158.

[17] Chelvam, T.T. and Asir, T., 2013. The intersection graph of gamma sets in the total graph of a commutative ring-I. *Journal of Algebra and Its Applications*, 12(04), p.1250198.

[18] Tehranian, A. and Maimani, H.R., 2011. A Study of the Total Graph. *Iranian Journal of Mathematical Sciences and Informatics*, 6(2), pp.75-80. http://ijmsi.ir/article-1-239-en.html

[19] West, D.B., 2001. *Introduction to graph theory* (Vol. 2). Upper Saddle River: Prentice hall.

[20] Wisbauer, R., 1996. *Modules and algebras: bimodule structure on group actions and algebras* (Vol. 81). CRC Press.

Mohammad Baziar
Department of Mathematics,
Yasouj University,
Yasouj Iran
Email: mbaziar@yu.ac.ir

Sakine Zare Khafri
Email: sa_zare66@yahoo.com