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LOCATING EMERGENCY AND RED CRESCENT CENTERS IN DEZFUL COUNTY USING GAME THEORY

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Abstract. Considering that if the number of emergency bases of the University of Medical Sciences or the Red Crescent is greater, then they will enjoy more budget and benefits from the government, and ultimately their employees will enjoy higher salaries, in this case a competition will be formed between the Red Crescent Organization and the University of Medical Sciences to have more relief bases.

The model considered in this paper takes into account competition between emergency services and the Red Crescent, as well as the impact of the distance of a facility on the provision of services, which is considered the most critical internal factor affecting the location of these facilities. In this study, two key factors the distance from injured or sick individuals and the significance of different zones are considered the primary criteria for location selection. The locations of the emergency centers of Dezful University of Medical Sciences and the Red Crescent centers across the 28 districts of Dezful County will be analyzed. Using game theory in a discrete space on a graph, 8 rounds of competition between the two organizations will take place. Each organization will aim to position its 4 facilities within the 28 districts of Dezful County in such a way that they maximize both coverage of demand and its own benefits. The proposed model in this study is a Voronoi game model, which enables the design of facility locations and arrangements in competitive markets.

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1. Introduction and preliminaries

One of the fundamental criteria for locating emergency centers is the average response time to incidents. Another important criterion for evaluating a disaster management system is the ability to respond to and handle incidents within a time frame shorter than the defined threshold and at a specified level of confidence. The significance of this indicator is such that in many strategic safety and disaster management programs, system efficiency is targeted and evaluated based on this criterion. Game theory is a branch of applied mathematics that studies strategic situations in which players choose different actions in an attempt to maximize their returns. The theory is widely used in economics, political science, psychology, as well as in logic and computer science. The foundational work in Game theory was developed by John von Neumann and Oskar Morgenstern in their book [17].

Strategic Games. A strategic game consists of a set of players, a set of strategies available to each player, and a payoff function that assigns a payoff to each player depending on the strategy chosen by each player.

Nash Equilibrium. A Nash equilibrium is a situation in a strategic game where no player can benefit by unilaterally changing their strategy, provided that the other players keep their strategies unchanged. This concept was introduced by John Nash in 1950 [11].

Mixed Strategies. In some games, players may choose to randomize over their possible strategies. Such a randomization is called a mixed strategy. A mixed strategy equilibrium occurs when players randomize in such a way that no player can improve their expected payoff by deviating from their mixed strategy.

Game Theory has a wide range of applications. In economics, it is used to model market competition, auctions, and bargaining. In political science, it helps analyze voting systems and coalition formation. In biology, it is applied to the study of evolution and population dynamics. Game Theory provides a powerful tool for understanding and predicting the behavior of agents in competitive situations. Its concepts are not only applicable to economics and politics but also to various fields such as biology, computer science, and beyond. The primary concern of managers in any organization is the optimal and efficient conversion of available capital resources into the maximum possible revenue

through investment in the right area, at the right time, and in the right place. Therefore, it can be said that location selection for urban services is one of the critical issues in today's world. The selection of appropriate locations for establishing service and production units has long been a focus for managers and researchers. The choice of location itself significantly impacts the costs of the production or service unit, as well as the accessibility to production resources, including transportation facilities, raw materials, and labor.

The difference between the mathematical models presented in game theory and operations research is that, in operations research, since the objective function and the problem constraints are fixed, optimization is conducted under certain conditions, and only the optimal solution is considered. However, in game theory, the options available to the players and the actions they take are known, but the outcome of each stage may differ from the next (similar to a game of chess). For this reason, game theory can be considered a form of optimization under uncertain conditions. Therefore, a game can be defined as a "mathematical model of optimal decision-making in collision (uncertain) conditions" [1].

Game theory is one of the advanced scientific phenomena with widespread applications across various fields, providing robust frameworks for strategic behavior. This theory analyzes the decision-making process of agents, whose decisions depend on the choices and behaviors of other competitors. Although the origins of this theory lie in economics, its use has expanded and found significant applications in various sciences [**?**]. This theory allows researchers to apply precise mathematical tools and methods to analyze and study the social, economic, and political relationships of humans and societies [5]. The application of game theory to location selection is also a topic of interest among researchers.

The aim of this research is to examine the maximum service delivery between Red Crescent and emergency centers in Dezful during holidays. In this study, for both organizations, location coverage is independently determined by considering factors such as the population of each area, the geographical area of each zone, expert opinions, and so on, to establish how many stations are necessary and in which parts of Dezful (28 zones) they should be placed to provide adequate coverage. For this purpose, using game theory and Nash equilibrium methods, the arrangement of each center's stations is determined so that each organization can cover different areas, serve the maximum number of visitors,

and achieve the highest level of service delivery. It is also assumed that visitors do not differentiate between the organizations and prefer to go to the nearest center.

2. Game Theory

Game theory models were first used in the 1950s in economic theory and political science. In the 1970s, game theory was initially used as a tool in evolutionary biology. As a result, the framework of game theory infiltrated microeconomic theory and found applications in many other branches of economics as well as a wide range of behavioral and social sciences. The 1994 Nobel Prize in Economic Sciences was awarded to three game theory theorists: John Harsanyi, John Nash, and Reinhard Selten. John Nash, one of the theorists in game theory, was employed by the Pentagon in the early 1950s, at the height of the Cold War, to combat Russian spies.

Kahneman, in an article related to the 1994 Nobel Prize in Economics, mentions that it is a historical fact that the main financial support for research in game theory came from the U.S. military. Aumann states that "the primary application of game theory was in military tactical issues, such as missile defense. Von Neumann was one of the key members of the Manhattan Project, a research and development project aimed at producing the first nuclear weapons, which took place between 1942 and 1946 in collaboration with the United States, the United Kingdom, and Canada.

Game theory uses mathematical methods to analyze decision-making in conflict situations. The goal of game theory is to determine the behavior of the player (decision-maker) so that the player's interests are optimized against the opponent's strategy.

In the existing research that studies competitive location problems, a game known as the Voronoi game is generally used, and will be discussed in detail later. In Voronoi games, the conditions for both players are entirely equal in terms of price and product quality, and customers choose the nearest service provider. Additionally, in Voronoi games, it is assumed that the distribution of players in the game area is uniform.

TABLE 1. Studies Conducted Based on Shortest Distance (Voronoi Game)

Space	Property	One Dimension	Two Dimensions
Line Segment	One-dimensional on a line segment	[?]	[?]
Circle	One-dimensional on a circular region	[8]	[8]
Multiple Circles	One-dimensional on multiple circles	[9]	[9]
Line Segment	Two-dimensional on a line segment	[?]	[?]
Two Dimensions	With higher dimensions	-	[16]
Empty Space	Two-dimensional	-	[16]

3. RESEARCH METHODOLOGY

To determine the optimal number of branches, expert opinions were applied. In order to solve the proposed model in this research, a Visual Basic software, designed and registered by the researchers, was utilized.

3.1. **Introduction of the Proposed Algorithm for Solving the Problem. n-Round Voronoi Game on Graphs:**

As previously mentioned, in most competitive location problems, the market area is two-dimensional. Since it is not possible to place services at any point in the space, the game area can be represented as graphs that discretely display the playing field. Thus, the game used under these conditions is called the discrete Voronoi game. For more information, refer to Sources[6, 12, 3, 18, 4].

In the discrete Voronoi game, the playing area is not a continuous region but a finite graph G, where two players, White and Black, alternately occupy an empty node on the graph over n consecutive rounds. Similar to chess, White starts the game. An empty node is a node that has not been previously occupied. Therefore, the two players cannot share a node simultaneously. It is assumed that the playing area G consists of at least $2n$ nodes [10, 13].

In this graph, each node receives the necessary services from the nearest node occupied by either company W or B, according to the nearest-neighbor rule. The score of each player is calculated based on the number of nodes they have covered. In cases where the distance from a node to nodes occupied by W and B is equal, an equal score is awarded to both players. At the end of the game, the player who controls the larger area (i.e., the most nodes) wins the game [10, 13].

The set of nodes transferred to player W_i from player B_i at the end of round 1, denoted by $W_i(B_i)$, is determined by the minimum distance from a node v to the occupied nodes. If there is no shortest path between the two nodes v and w, $d(v, w) = \infty$ is considered, indicating that there is no direct path between the two nodes. The set of nodes transferred by W or B is defined as follows:

$$V(A, B) = \{u \in V \mid \min_{v \in A} d(u, v) < \min_{w \in B} d(u, w)\}$$

Note that here $d(.,.)$ refers to the shortest path distance between two vertices along the edges of the graph.

The dominant sets $V(B_i, W_i)$ and $V(W_i, B_i)$ represent, respectively the sets of nodes dominated by players B and W at the end of round 1. The sets $V(B_n, W_n)$ and $V(W_n, B_n)$ represent the final dominated nodes after n rounds, as some nodes will be closer to player W or B. The nodes dominated by B_i and W_i are shown as $V(W_i, B_i)$ and $V(B_i, W_i)$, respectively.

Since some nodes are equidistant from the nodes occupied by W and B, set N_i is used to represent neutral nodes, which is shown below, and the elements of this set do not belong to any of sets $V(B_i, W_i)$ and $V(W_i, B_i)$.

$$N_i := \{u \in V \mid \min_{w \in W_i} d(u, w) = \min_{v \in B_i} d(u, v)\}$$

In the final stage of the game, the player who controls the most nodes wins. This is determined by comparing the number of nodes dominated by W and B. If $|V_W| > |V_B|$, then the white player W wins, and if $|V_B| > |V_W|$, then the black player B wins. If $|V_W| = |V_B|$, the game ends in a draw.

Figure 1 illustrates a graph showing that the larger circles represent the nodes dominated by players W and B, where the smallest circles indicate the proximity of the dominated nodes.

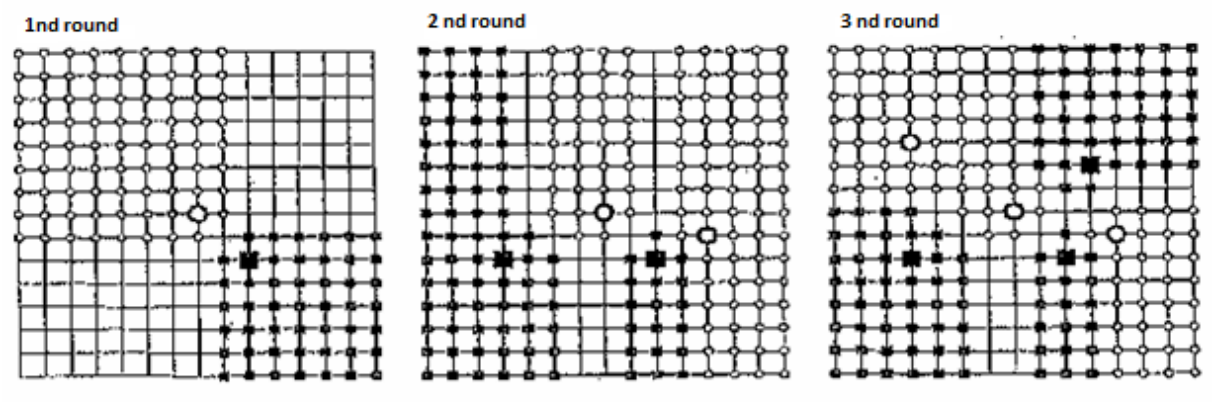


FIGURE 1. An example of a 15x15 game environment graph

In this section, we consider the case where the game area G is a k-ary rooted tree where all the internal nodes have exactly k children and all the leaves are at the same level. This type of Voronoi game, denoted by $VG(T, n)$, where $2n \leq k$, is a simple version of Voronoi games. In this game, the white player moves first, occupying the root T, and with this action, controls at least $\frac{n-1}{k}$ nodes. From that point on, the black player controls at most $\frac{n-1}{k}$ nodes, meaning that the white player wins the game. The winning strategy for the white player is provided in the algorithm [10, 13].

In the above algorithm, the white player occupies the empty root. Any game area T that satisfies the condition $2n \leq k$ represents the winning strategy for the white player.

Consider the Voronoi game $VG(G, n)$ where G is a k-ary tree with $2n \leq k$. In this case, if the white player moves first, they will always win.

If the k-ary tree is a complete binary tree and T is a k-ary tree with height H and number of nodes N, we have:

$$H \sim \log_k N, \quad N = \frac{k^{H+1} - 1}{k - 1}$$

For this game, it can be proven that the following proposition holds:

Proposition. The generalized Voronoi game $VG(G, N)$ on a k-ary tree G with $N \geq 4n^2$. If the white player wins the game, then despite the conditions in which the players act most optimally, the game will still end in a draw [10, 13].

Of course, different conditions can result in different outcomes, where each of these cases can be considered. Specifically, if the numbers of white and black pieces are equal, the analysis of the Voronoi game for k-ary trees requires separate investigation.

This well-known Voronoi model is recognized and widely used to solve various game theory problems, such as the method of solving for two pieces of different colors in a two-dimensional space. In this way, it is possible to resolve different situations like red-blue games, where if the number of pieces, say 4.5×10^5 pieces, and the initial position are well defined, the problem can be solved manually. The algorithm designed in this research [8] and implemented using Visual Basic can compute with great accuracy considering the distance and importance of each node, allowing the player to determine the winner at each stage. This software has been registered by the researchers in the Scientific Deputy of the Republic.

4. Findings

In this research, data from the Governor's Office and the Municipality of Dezful County were used to determine the population, area, and zones of Dezful. Dezful County consists of two main regions, and these regions are further divided into smaller zones from different perspectives, such as municipal zoning, tourism, and public transportation. In this study, the zones were thoroughly examined to ensure that all units were relatively consistent. To determine the optimal number of stations for Red Crescent and emergency services in Dezful County, expert opinions, including officials and staff from these organizations, were utilized. A total of 45 individuals were interviewed to determine that a minimum of four stations was required in the county. The location of these stations were determined based on demand, the number of requests, and the prevailing conditions. Figure 2 shows a map of Dezful County with the zoning of its areas indicated by letter h, and the importance factor of each zone indicated by letter z.

To determine the importance factor of each zone, we have used the Analytic Hierarchy Process (AHP) or Weighted Sum method. To do this, we considered factors such as
1- Population and area (determining the demand density and geographical size of each region using provincial and municipal data),
2- Historical demand (the number of requests and needs recorded for emergency and Red Crescent services in the past),

3- Current conditions (qualitative and infrastructure factors such as the existence of critical infrastructure, access roads, or accident-prone areas).

Method AHP is one of the most common methods in location studies that requires expert opinions. The following steps have been taken for this:

1- Normalization: First, the objective values (population, number of requests, etc.) of each zone were normalized to a common scale (e.g., between 0 and 1).

2- Factor Weighting: Through interviews with municipal experts and ordinary people in each zone (45 people were interviewed), the relative weight of each factor was determined. For example, experts in one zone determined that "historical demand" was 0.60 important, while "population" was 0.40 important.

3- Calculation of the final coefficient: The final importance coefficient z_i for each zone i is obtained from the sum of the weights of the normalized factors:

$$z_i = (a_{P_i} \times P_i) + (a_{D_i} \times D_i) + (a_{C_i} \times C_i)$$

where,

a_{P_i}: Population Importance Percentage in zone i,

P_i: Population of region i,

a_{D_i}: Demand Importance Percentage in zone i,

D_i: Demand in zone i,

a_{C_i}: Percentage of importance of conditions in zone i,

C_i: conditions in zone i.

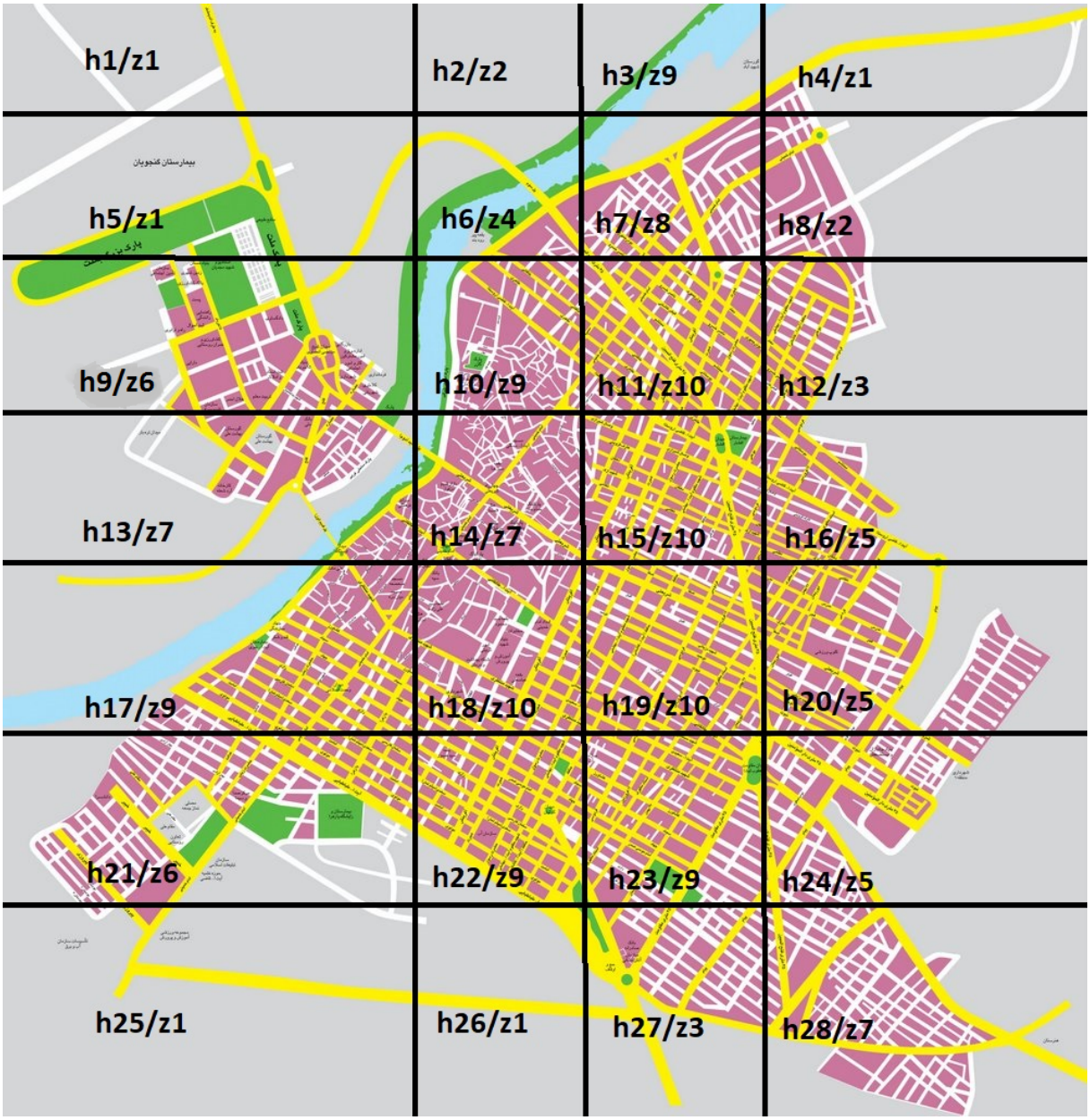


FIGURE 2. Map of Dezful County: Zones and the Importance Factor of Each Zone

5. Problem Solving

In the graph $G(V, E)$, to model this problem for locating emergency and Red Crescent bases, the vertices represent one of the 28 zones that need to receive services from the nearest center. Edges also indicate the possibility of moving or transferring services

between two adjacent points in a discrete environment. More importantly, the edges are used to define distance. The distance that is considered in a discrete Voronoi game is usually the shortest path distance between two vertices along the edges of the graph.

In this section, for simplicity, we denote the Red Crescent by R and the emergency bases by M.

A player's final score is the sum of the importance factors of all the zones or vertices that the player has covered (coverage means the vertices that have the shortest path to the player's selected vertex). Also, each $v_{tr} \in N_i$ gives half of its importance factor to each of the players R and M.

Thus, if we represent the profit or scoring function with u, the final score of player R is calculated using the importance factor of z_{v_R} as follows:

$$u(R) = \sum_{v_R \in V(R_i, M_i)} z_{v_R} + 0.5 \sum_{v_{tr} \in N_i} z_{v_{tr}},$$

The final score of player M is also calculated using the importance factor of z_{v_M} as follows:

$$u(M) = \sum_{v_M \in V(M_i, R_i)} z_{v_M} + 0.5 \sum_{v_{tr} \in N_i} z_{v_{tr}}.$$

Let's assume that the Red Crescent, as the game starter, creates a station in Dezful City, established in zone 18 which has a weight of 10. At this point, the Red Crescent receives 160 points, while the emergency services have not yet received any points. Therefore, the total points that the stations can earn by the end of the first round, taking into account all the regions under the Red Crescent's control, is 160 points. This is because all visitors in these regions are now served by the Red Crescent station.

In the second round, the emergency base must decide in which area to build its first base, considering the importance coefficients. The emergency base builds its first base in Zone 19. At this stage, the Red Crescent receives 73 points, while the emergency base receives 87 points. In the next round, the Red Crescent builds its second base in Zone 15. At this stage, the Red Crescent receives 112 points, while the emergency base's score is 39. The game continues in the same manner. In the last round, when the emergency base must locate its base, since it has all sides and it knows that the game will end after this move (according to the problem assumption), it chooses the best area and at the end of this round, the emergency base wins the game with a total of 81.5 points from the

Red Crescent with a score of 78.5. At the end of the game, each of the two players has established four bases in the 28 districts of Dezful City.

The result of the game at the end of each round, the selected base area of the players at the end of each round, and the points earned by the players are presented in Table 2. In some cases, the total score of both players exceeds 160, this is because applicants in some areas use both service centers jointly according to the game rules, and therefore the points associated with those areas are considered for both the Red Crescent and the emergency base.

TABLE 2. Locations and Scores of the Stations in the Eight Rounds of the Game

Round	**Zone Red Crescent**	**Score Red Crescent**	**Zone Emergency Services**	**Score Emergency Services**
Round 1	18	160	-	-
Round 2	-	73	19	87
Round 3	15	112	-	48
Round 4	-	70.167	9	89.833
Round 5	6	97.166	-	62.834
Round 6	-	74	7	86
Round 7	23	98	-	62
Round 8	-	78.5	17	81.5

6. CONCLUSION

The model presented in this study is a Voronoi game model that allows designing location selection and service layout in competitive spaces. This model is designed for real-world scenarios in which proximity to customers is very important for providing services. Greater proximity to customers leads to attracting more customers and thus reducing transportation costs. In this study, the element of distance from the customer and the importance coefficient of each area were considered. The main factors in the selection of locations for Red Crescent and emergency service stations in 28 districts of Dezful City were analyzed using game theory in a graph-based space. Through this method, it was determined that each center (emergency or Red Crescent) can establish its base in

four districts out of 28 districts of Dezful City to maximize service provision and cover the highest demand. The results show that starting the game earlier and with an equal number of stations increases the probability of winning, as it allows the Red Crescent center to establish more bases and gain an advantage. If a player starts the game, he has a higher probability of winning the game because the second player has to take more actions to compensate for the first player's advantage. This explanation explains why the second player who starts later may be at a disadvantage because the first player has already covered a significant portion of the areas. Furthermore, the results show that the number of areas covered by a player and the priority given to each area, based on the importance coefficient, can affect the final outcome. This explanation explains why in this study, the number of bases increased significantly with the addition of a new area. This research can be particularly useful for various industrial and commercial activities operating in competitive environments. These activities usually aim to attract the highest profits and often compete to be as close as possible to customers.

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